Radioactivity and

Production of Radionuclides

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Decay of Radioactivity & Radionuclide Production

- Activity
- Exponential decay
- Half-life
- Specific activity
- Parent-daughter mixtures and radionuclide generators
    - secular equilibrium
    - transient equilibrium
    - no equilibrium
- Production mechanisms
    - neutron activation
    - nuclear fission byproducts
    - accelerator-produced

Activity = number of nuclei that decay per unit time

As t increases, N (# undecayed nuclei) and A decreases

Radioactivity (ABR core study guide 17.c.i(a)-(b))

Consider a sample of radioactive material. The fractional change in the number of radioactive atoms during some short time, Δt, is linearly related to the time interval. The constant of proportionality is called the **decay constant** for the radionuclide:

S*-1
Probability of a nucleus decaying per unit time

$$\frac{\Delta N}{N} = -\lambda \Delta t$$

$$Activity(Bq) = \left|\frac{\Delta N}{\Delta t}\right| = \lambda N,$$ where 1 Bq = 1 decay/second

Bq = becqueral

$$Activity(Ci) = \lambda N / (3.7x10^{10})$$ (Ci = Curies)

1 mCi = 3.7 x 10^7 dps = 37 MBq

Exponential Decay (ABR core study guide 17.c.i(b) and 17.c.iii)

Use calculus to solve for the number of radioactive atoms remaining in the sample as a function of time. (Integrate both sides of equation.)

$$\frac{dN}{N} = -\lambda dt$$

$\ln(N) - \ln(N_o) = -\lambda t$,

where N_o = initial number

$\ln(N/No) = -\lambda t$

$N(t) = N_o e^{-\lambda t}$

This proof shows that radioactive decay is exponential decay ---> this leads to the half-life concept

$A = \lambda N$

Thus:

$A(t) = A_o e^{-\lambda t}$

Half-Life (ABR core study guide 17.c.i(b) and 17.c.iii)

The half-life is the time required for the radioactivity to decay to half of its initial value:

$$\frac{1}{2} A_o = A_o e^{-\lambda t_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda t_{1/2}}$$

$$\ln(1/2) = -\lambda t_{1/2}$$

$$\ln(2) = \lambda t_{1/2}$$

$$t_{1/2} = \ln(2)/\lambda \sim 0.693/\lambda$$

For a fast rate of decay :
lambda is large, t(1/2) is short

(The **average** lifetime $= 1/\lambda$.)

Specific Activity and Tracer Principle (ABR core study guide 17.c.ii)

The **specific activity** is the ratio of the radioisotope's activity to the total mass of the same element or compound (Bequerels per gram).

The **carrier-free specific activity (CFSA)** is the highest possible specific activity of a radionuclide, i.e. with no "cold" carrier present.

CFSA is inversely related to half-life

CFSA(Bq/g) ~ $4.8 \times 10^{18}/(A\ t_{1/2})$, where

A = mass number of the radionuclide or compound,

$t_{1/2}$ = half-life in days.

(Note: Easier to get high specific activity for short half-life nuclides.)

CFSA(Ci/g) ~ $1.3 \times 10^{8}/(A\ t_{1/2})$, in old units.

<u>Specific Activity and Tracer Principle</u> (ABR core study guide 17.c.ii)

<u>Requirements of ideal tracers</u>:

1.Tracer behavior should be as close as possible to that of the natural substance

2.Mass of tracer should not alter underlying physiologic process

- rule of thumb: mass of tracer < 0.01 x mass of endogenous compound

3. Specific activity high enough to permit imaging or blood counting without violating conditions 1 and 2.

4. Any isotope effect should be negligible (or quantitatively predictable).

Example: What is the mass of 10 mCi of $H_2{}^{15}O$? (typical activity injected for PET)

- $t_{1/2}$ of ^{15}O is 2 minutes = 0.001389 days, and the molecular weight of $H_2{}^{15}O$ is 17.
- CFSA = 1.3×10^8 / (17 x .001389) = 5.5×10^9 Ci/g.
- A more typical specific activity might be 10% of the CFSA ~ 5.5×10^8 Ci/g.
- 10 mCi = 0.01 Ci, so mass = 0.01 Ci / 5.5×10^8 Ci/g = 18.2×10^{-12} g = 18.2 pg.
- **18.2 pg -- diluted throughout the whole body -- is clearly a trace amount.**

Radionuclide Equilibrium (Parent-Daughter Mixtures) (ABR core study guide 17.c.iv)

Complicated situation: parent radionuclide gives rise to new daughter radioactivity, even as the daughter's activity decays.

Activities are described completely by **Bateman equations.**

Approximations of interest:

- **secular equilibrium** ($T_p >> T_d$), e.g. Ra-226->Rn-222

  $$A_d(t) \approx A_p(t)(1-e^{-\lambda t})xB.R.$$ (1620 y >> 4.8 d)

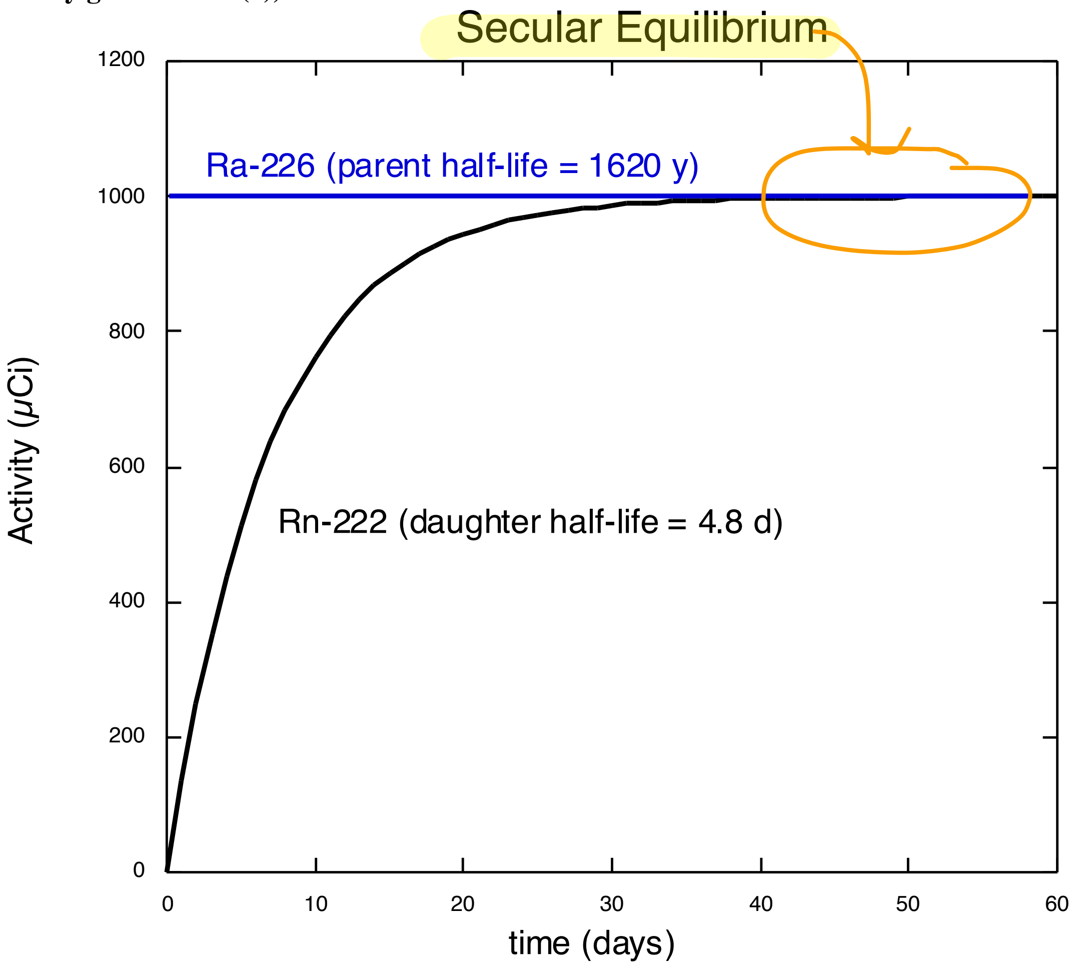
- **transient equilibrium** ($T_p > T_d$), e.g. Mo-99 -> Tc-99m

  $$\frac{A_d}{A_p} = \left[\frac{T_p}{T_p - T_d}\right]xB.R.$$ (66 h > 6 h)

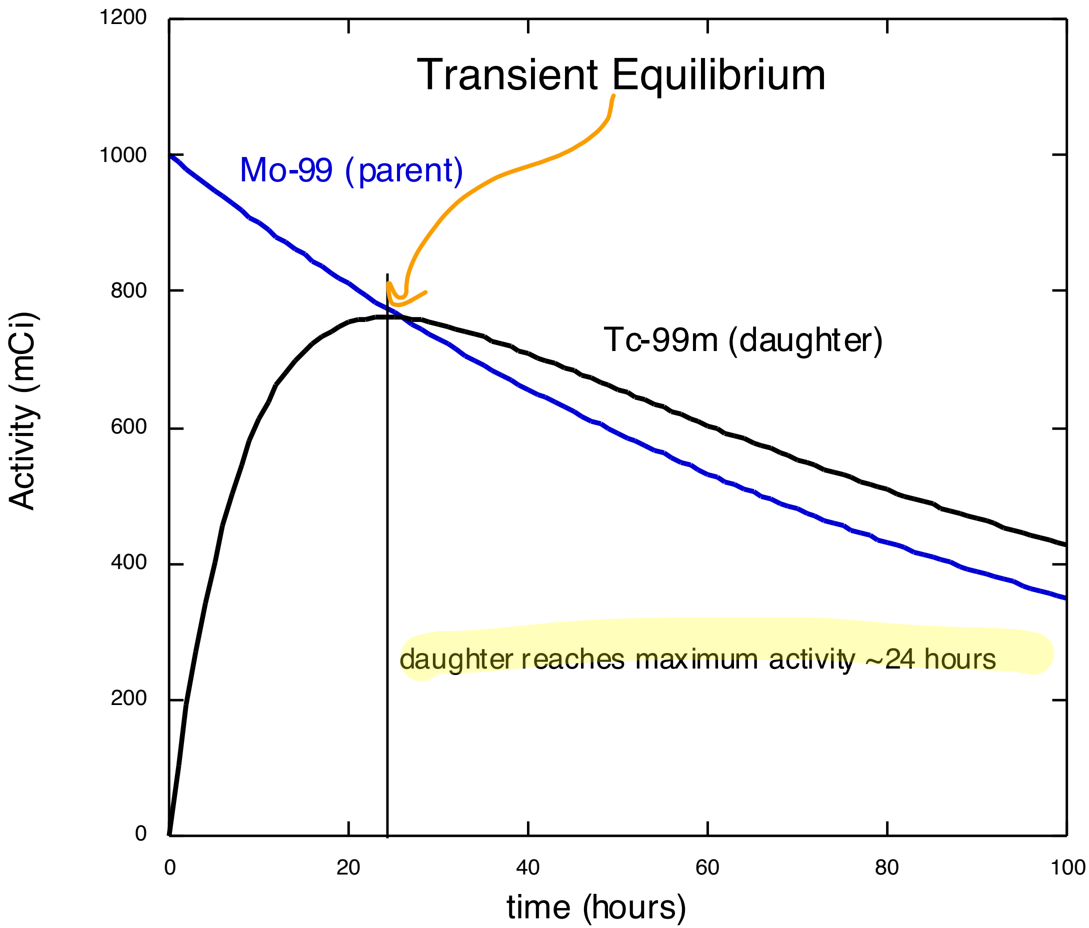
- **no equilibrium** ($T_d > T_p$), e.g. Te-131m -> I-131

  parent goes away, daughter decays (30 h < 8 d)

(ABR core study guide 17.c.iv(a))



(ABR core study guide 17.c.iv(a))



(ABR core study guide 17.c.iv(b))

Mo-99 / Tc-99m Generator
(transient equilibrium)

inject saline

Molybdate ions, $^{99}MoO_4^{2-}$ bound to an alumina (Al_2O_3) column.

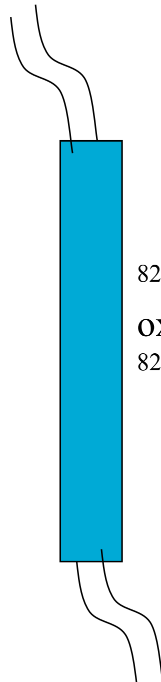
^{99}Mo $t_{1/2}$=66 h; ^{99m}Tc $t_{1/2}$=6 h

Collect $^{99m}TcO_4^-$ (pertechnetate) in eluate

- Check for alumina (<10 μg/mL) and Mo-99 breakthrough (<0.15 μCi Mo-99 / mCi Tc-99m)

Sr-82 / Rb-82 Generator
(secular equilibrium)

inject saline



^{82}Sr adsorbed on a stannic oxide (SnO_2) column.

^{82}Sr $t_{1/2}$ = 25.6 d; ^{82}Rb $t_{1/2}$ = 75s

Collect $^{82}RbCl$ in eluate

- Check Sr-82<.02kBq and Sr-85<.2kBq per MBq of Rb-82 administered

(ABR core study guide 17.d.i)

Radionuclide Production: Neutron Activation

- In a nuclear reactor, fission reactions break apart U-235 into multiple "fission fragments" and release lots of neutrons.

- The neutrons can be used to irradiate various targets, which are placed inside the reactor. The targets absorb neutrons to become "activated".

1. P-32 production (14.3 day $t_{1/2}$)

$$^{31}P(n,\gamma)^{32}P$$

2. Cr-51 production (27.8 days $t_{1/2}$)

$$^{50}Cr(n,\gamma)^{51}Cr$$

(ABR core study guide 17.d.ii)

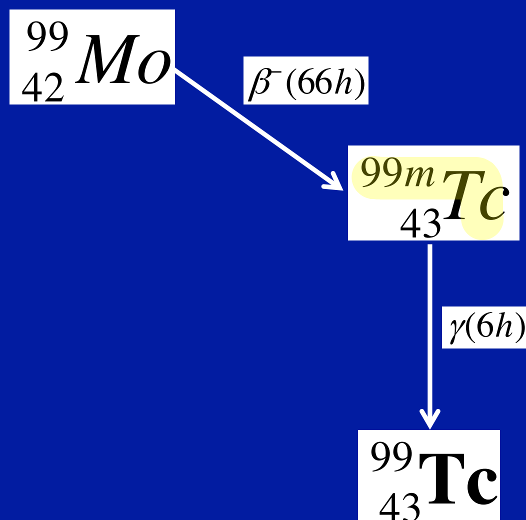
Radionuclide Production: Fission Byproducts

1. Generator Production of Tc-99m (many uses)

In nuclear reactor:

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{99}_{42}Mo + ^{134}_{50}Sn + 3^{1}_{0}n + \gamma + \sim 200\,MeV$$

In generator:



(ABR core study guide 17.d.ii)

Radionuclide Production: Fission Byproducts

2. I-131 Production (used for thyroid imaging + therapy):

$$ {}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{131}_{53}I + {}^{102}_{39}Y + 3\,{}^{1}_{0}n + energy $$

$$ {}^{131}_{53}I \xrightarrow{\beta^-\ (8d)} {}^{131}_{54}Xe $$

3. Xe-133 is another fission byproduct (lung vent imaging)

(ABR core study guide 17.d.iii)

Cyclotron: Principle of Operation

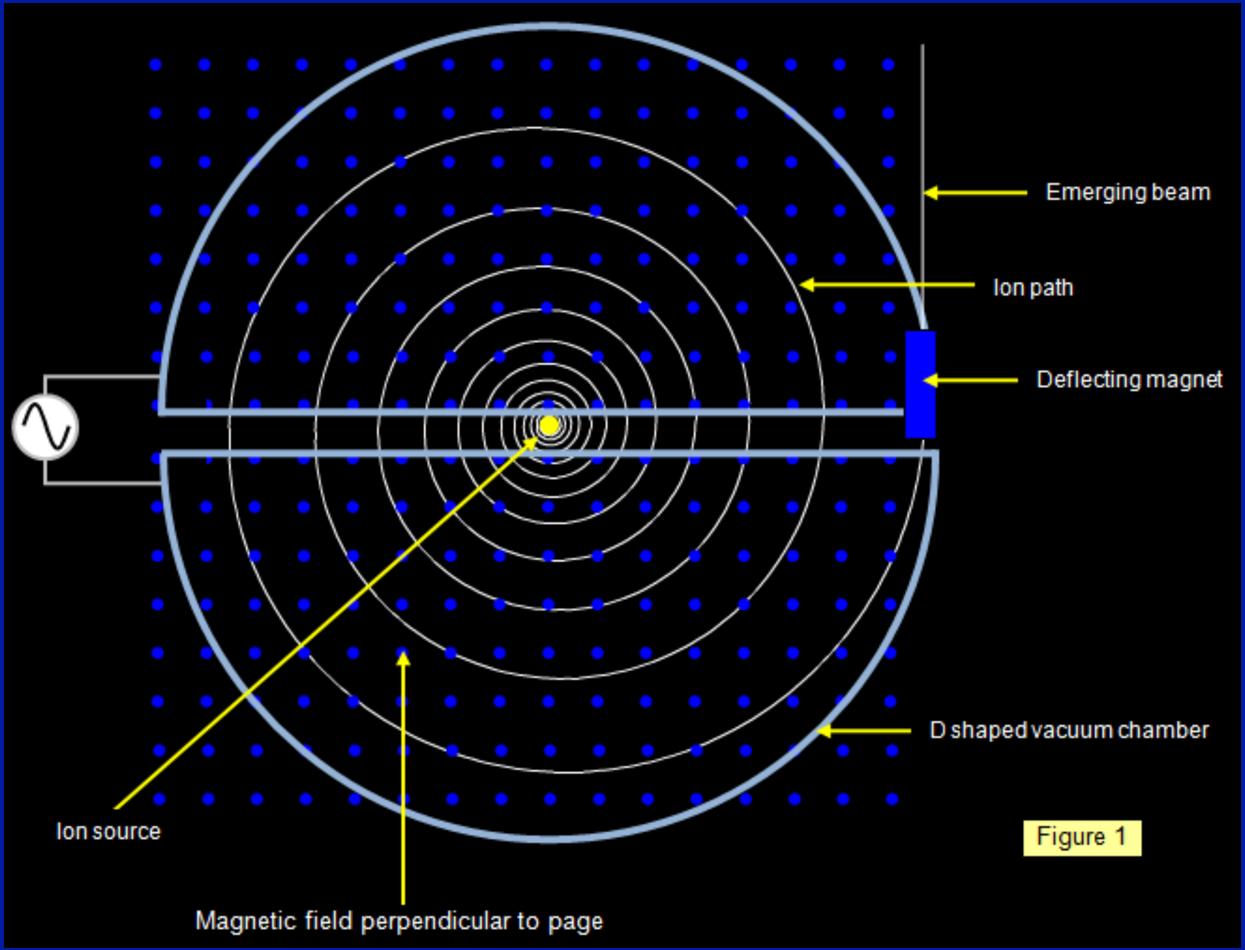


Diagram from http://www.schoolphysics.co.uk

(ABR core study guide 17.d.iii)

Radionuclide Production: Cyclotron Produced

1. Thallium-201:

$$^{203}\mathbf{Tl(p,3n)}^{201}\mathbf{Pb}$$

$EC(9.4h)$

$$^{201}Tl$$

$EC(73h)$

$$^{201}Hg$$

Image mercury x-rays

- myocardial perfusion
- tumor imaging

(ABR core study guide 17.d.iii)

Radionuclide Production: Cyclotron Produced

2. I-123:

- thyroid imaging
- MIBG imaging

$$^{124}\mathbf{Xe(p,2n)}^{123}\mathbf{Cs}$$

$\beta^{+}(5.87m)$

$$^{123}Xe$$

$EC(73h)$

$$^{123}I$$

$EC(13.2h)$

$$^{123}Te$$

(ABR core study guide 17.d.iii)

Radionuclide Production: Cyclotron Produced

3. In-111:

- octreotide
- WBCs

$$^{111}\mathrm{Cd(p,n)}^{111}\mathrm{In}$$

$$^{111}\mathrm{In} \xrightarrow{EC(67h)} {}^{111}Cd$$

4. Ga-67:

- lymphoma and
- infection imaging

$$^{66}\mathrm{Zn(d,n)}^{67}\mathrm{Ga}$$

$$^{67}\mathrm{Ga} \xrightarrow{EC(78h)} {}^{67}Zn$$

(ABR core study guide 17.d.iii)

Radionuclide Production: Cyclotron Produced PET Tracers

5. F-18 (e.g., FDG): $^{18}O(p,n)^{18}F$ (110 min)

6. N-13 (e.g., ammonia): $^{16}O(p,\alpha)^{13}N$ (10 min)

7. O-15 (water) $^{14}N(d,n)^{15}O$ $^{15}N(p,n)^{15}O$ (2.0 min)

8. C-11 (e.g, acetate) $^{14}N(p,\alpha)^{11}C$ (20.4 min)